CALCULATION OF RESIDUAL OIL SATURATION IN AN INHOMOGENEOUS BED IN THE CASE OF MUTUAL DISPLACEMENT OF FLUIDS WITH VISCOPLASTIC PROPERTIES

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In connection with the existence of petroleum having a limiting shear stress under stratal conditions [1], it is of interest to estimate the residual oil saturation in the case of its displacement from a porous medium by fluids with viscoplastic properties. The effect of the viscoplastic properties of fluids and capillary and hydrodynamic forces on the distribution of residual saturation in an inhomogeneous bed was investigated for the example of a linear bed consisting of two sections of different permeability. The equations of motion for each phase are written with consideration of the limiting gradient of shear stress. The calculations are carried out on the basis of the relationships obtained. The results of the calculations are discussed.

Let one viscoplastic fluid displace another in a horizontal bed of length L consisting of two sections. On each section the absolute permeability k_i and porosity m_i are constant but change abruptly on passing through the boundary of the sections ($i = 1, 2$ is the number of the section read from the entrance to the bed).

One-dimensional percolation of two immiscible and incompressible viscoplastic fluids can be described by a Darcy-type equation with correction for the shear-stress gradient [2]. In the general case the relative phase permeabilities and capillary pressure are supposed dependent on saturation, flow velocity, and rheological properties of the fluids:

$$v_{ji} = -k_i \frac{k_{ji}(\rho_i, \pi_i)}{\mu_j}\left(\frac{\partial p_{ji}}{\partial x} + \tau_{ji}\right), \quad \pi_i = \frac{k_i \Delta\tau_i}{v\mu_1}, \quad p_{ci}(\rho_i, \pi_i) = p_{1i} - p_{2i} \tag{1}$$

Here x is a coordinate in the direction of movement; v_{ji} is the velocity of percolation of phase j in section i; μ_i is viscosity; k_{ji} is the relative phase permeability; ρ_i is saturation of the medium by the fluid being displaced; τ_{ji} is the gradient of shear stress; π_i is a dimensionless parameter expressing the relationship of the plastic forces and hydrodynamic forces; $\Delta\tau_i = \tau_{2i} - \tau_{1i}$; $V = V_{1i} + V_{2i}$ is the total percolation velocity; p_{ji} is the pressure in the phase; p_{ci} is the capillary pressure. It is considered that the index $j = 1$ pertains to the displacing and $j = 2$ to the displaced fluid.

To determine the residual saturation [3] of the displaced phase, we set $V_{2i} = 0$ in (1). This can be achieved in two cases:

$$k_{2i} = 0 \quad \text{or} \quad \frac{\partial p_{2i}}{\partial x} + \tau_{2i} = 0$$

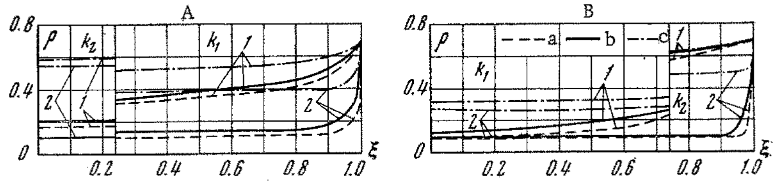

Fig. 1

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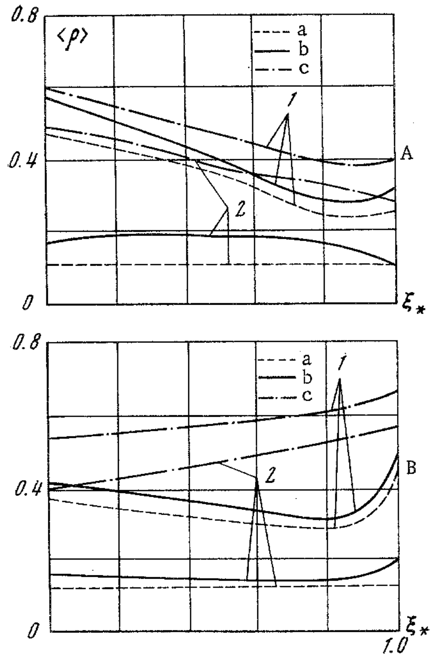

Fig. 2

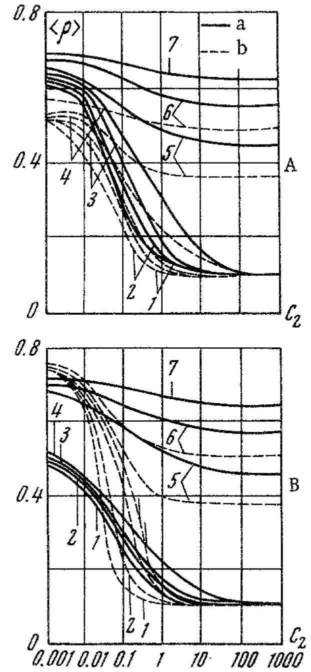

Fig. 3

We will consider the second case, assuming V = const. Then we obtain from (1) the following problem:

$$\frac{d\rho_i}{dx} = \frac{1}{dp_{ci}/d\rho_i}\left[\Delta\tau_i - \frac{v\mu_1}{k_i k_{1i}(\rho_i)}\right] \tag{2}$$

$$\rho_2 = \rho_0,\ x = L;\ p_{c1}(\rho_-) = p_{c2}(\rho_+),\ x = l \tag{3}$$

The boundary conditions (3) establish constant saturation at the exit from the bed and continuity of the capillary pressure at the interface of the sections of the bed (ρ_-, ρ_+ are saturation at the left and right interfaces of the sections, respectively).

We introduce the variable $\xi = x/L$ and the Leverett function

$$\varphi_i(\rho_i) = \frac{p_{ci}}{\sigma}\left(\frac{k_i}{m_i}\right)^{1/2}$$

(σ is interphase tension). We transform Eq. (2) and conditions (3) to a dimensionless form

$$\frac{d\rho_i}{d\xi} = -\frac{C_i}{\varphi_i'(\rho_i)}\left[\varkappa_{1i}(\rho_i) - \pi_i\right] \tag{4}$$

$$\rho_2 = \rho_0,\ \xi = 1$$
$$\varphi_1(\rho_-) = \left(\frac{k_1 m_2}{k_2 m_1}\right)^{1/2}\varphi_2(\rho_+),\ \xi = \xi_* = l/L \tag{5}$$

$$\varphi_i' = \frac{d\varphi_i}{d\rho_i} < 0,\ C_i = \frac{v\mu_1 L}{\sigma\sqrt{k_i m_i}},\ \varkappa_{1i}(\rho_i) = \frac{1}{k_{1i}(\rho_i)} \tag{6}$$

As follows from (1), the dimensionless parameter π_i can be positive, equal to zero, and negative. Equation (4) can formally have three solutions of $\rho_i(\xi)$, for which

$$d\rho_i/d\xi < 0,\ d\rho_i/d\xi = 0,\ d\rho_i/d\xi < 0$$

If $\pi_i \le 0$, then

$$\varkappa_{1i}(\rho_i) - \pi_i > 0,\ d\rho_i/d\xi > 0$$

i.e., residual saturation increases toward the exit from each section. When $\pi_i > 0$, the condition

$$\varkappa_{1i}(\rho_i) - \pi_i \geqslant 0 \tag{7}$$

should be observed, since otherwise $d\rho_i/d\xi < 0$, which is without physical meaning.

The root $\rho_i = \rho_i^\circ$ of equation $\varkappa_{1i}(\rho_i) - \pi_i = 0$ represents the value of the limiting residual saturation for a given value of the parameter $\pi_i > 0$. A uniform distribution ($d\rho_i/d\xi = 0$) is physically possible, if at the exit of each section a saturation equal to ρ_i° is established. For all $\rho_i > \rho_i^\circ$ we will have integral curves of $\rho_i(\xi)$ with derivatives $d\rho_i/d\xi > 0$, which are of principal interest.

The foregoing and the experimental investigations [1] indicate that the phase permeabilities and the capillary pressure for viscoplastic fluids depend on the parameter π.

However, presently there are no reliable data on the empirical functions $k_1(\rho, \pi)$, $\varphi(\rho, \pi)$. Therefore, on the basis of the known concepts of the character of curves $k_1(\rho)$ and $\varphi(\rho)$, in the calculations under consideration the indicated dependences can be selected analogous to those found for viscous fluids, in which case relationship (7) should be fulfilled for the given values of π.

The solution of problem (4), (5) gives the distribution of residual saturation respectively for the second and first sections of the bed

$$\xi = 1 - \frac{1}{C_2}\int_{\rho_0}^{\rho_2} \frac{\varphi_2'(\rho_2)\, d\rho_2}{\varkappa_{12}(\rho_2) - \pi_2}, \quad \xi = \xi_* - \frac{1}{C_1}\int_{\rho_-}^{\rho_1} \frac{\varphi_1'(\rho_1)\, d\rho_1}{\varkappa_{11}(\rho_1) - \pi_1} \tag{8}$$

From distributions (8) we calculate the weighted-mean residual saturation, which characterizes the final yield for the sections and for the bed as a whole

$$\langle\rho_1\rangle = \frac{1}{\xi_*}\int_0^{\xi_*} \rho_1(\xi)\, d\xi, \quad \langle\rho_2\rangle = \frac{1}{1-\xi_*}\int_{\xi_*}^{1} \rho_2(\xi)\, d\xi, \quad \langle\rho\rangle = \langle\rho_1\rangle \xi_* + \langle\rho_2\rangle (1 - \xi^*) \tag{9}$$

The residual saturation was calculated by means of relationships (8), (9) on the "Nairi" computer for a wide range of variation of parameters C_2 and π_2, their relation with parameters C_1 and π_1 from (6), (1) being used

$$C_2 = \left(\frac{k_1 m_1}{k_2 m_2}\right)^{1/2} C_1, \quad \pi_2 = \frac{k_2 \Delta\tau_2}{k_1 \Delta\tau_1}\pi_1 \quad \text{or} \quad \pi_2 = \left(\frac{k_2}{k_1}\right)^{1/2}\pi_1$$

The last relationship is obtained if the gradient of shear stress is expressed in terms of the limiting shear stress [1].

In the calculations we took $k_2/k_1 = 10$, $m_2/m_1 = 2$, and $\rho_0 = 0.7$.

The relative permeabilities of the first phase $k_{1i}(\rho_i)$ for the nonwetting fluid on both sections of the bed, $k_{12}(\rho_2)$ for the wetting fluid on the high-permeable section, and $k_{11}(\rho_1)$ for the wetting fluid on the low-permeable section were assigned respectively in the following form:

$$k_{1i}(\rho_i) = \left(\frac{0.9 - \rho_i}{0.8}\right)^3, \quad k_{12}(\rho_2) = \left(\frac{0.9 - \rho_2}{0.9}\right)^3, \quad k_{11}(\rho_1) = \left(\frac{0.88 - \rho_1}{0.9}\right)^3$$

The capillary functions in cases of displacement by a nonwetting fluid and displacement by a wetting fluid had respectively the following form:

$$\varphi_i(\rho_i) = \frac{0.9072}{\rho_i - 0.09} - 9.2123\,\rho_i^{20}, \quad \varphi_i(\rho_i) = 9.2123\,(1 - \rho_i)^{20} - \frac{0.9072}{0.91 - \rho_i} \quad (\pi_i \leqslant 0)$$

$$\varphi_i(\rho_i) = \frac{0.9072}{\rho_i - \rho_i^\circ} - \delta_i \rho_i^{20}, \quad \varphi_i(\rho_i) = \beta_i (1 - \rho_i)^{20} - \frac{0.9072}{0.9 - \rho_i} \quad (\pi_i > 0)$$

$$\delta_i = \frac{17.462}{0.9 - \rho_i^\circ}, \quad \beta_i = \frac{0.9072}{(1 - \rho_i^\circ)^{20}(0.9 - \rho_i^\circ)}$$

We found the distributions and value of the weighted-mean residual saturation and dependences on the location of the more or less permeable sections in the direction of percolation, their relative size, wettability of the porous medium, and relationship of hydrodynamic and plastic forces.

Figure 1 shows the distribution of residual saturation for displacement by a nonwetting (A) and wetting (B) fluid, where curves 1 and 2 correspond to values $C_2 = 0.1$ (curves a, b, and c correspond to values $\pi_2 = -10$, 0, and 10). We see from Fig. 1 that at the interface of the sections there is a discontinuity of residual saturation, its value being greater on the side of the high-permeable section than on the side of the low-permeable when $\pi_2 > 0$ (the second phase has a greater shear stress than the first). In the case $\pi_2 < 0$ (the first phase has a greater shear stress than the second) the opposite picture is observed, if the displaced phase selectively wets the rocks better. With an increase of C_2 the effect of inhomogeneity of the porous medium on the distribution of residual saturation decreases.

The weighted-mean residual saturation $<\rho>$ is shown in Fig. 2 as a function of the parameter ξ_* and direction of percolation (A is injection of the wetting fluid from the low-permeable section, and B is injection of nonwetting fluid from the high-permeable section, otherwise the legend is the same as in Fig. 1).

With a certain combination of the parameters of the process a maximum complete replacement of one fluid by another can be obtained. The completeness of displacement decreases with an increase of $\pi_2 > 0$.

Figure 3 gives the weighted-mean residual saturation as a function of C_2 and π_2 for the value $\xi_* = 0.5$ when displacement is accomplished from the low-permeable (A) and high-permeable (B) section by a nonwetting (a) and wetting (b) fluid. The following values of π_2 correspond to the curves:

Curves	*1*	*2*	*3*	*4*	*5*	*6*	*7*
$\pi_2 =$	−50	−25	−10	0	10	25	50

An analysis of the dependences shows that the volume of the undisplaced fluid decreases with increase of C_2, and for each value π_2 = const there exists a value of C_2 such that beginning with which a further increase of this parameter changes $<\rho>$ little. The latter is manifested at small values of C_2, when the displacing phase is wetting. If the less permeable section is located at the entrance to the bed, more wetting fluid remains than nonwetting. When the high-permeable section is located at the entrance, the effect of wettability on the final yield depends on C_2.

The effectiveness of displacement by viscoplastic fluids increases with the absolute value of the parameter $\pi_2 < 0$.

LITERATURE CITED

1. R. S. Gurbanov, A. F. Kasimov, and A. Kh. Mirzadzhanzade, "Hydrodynamics of viscoplastic media," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 3 (1967).
2. M. G. Bernardiner and V. M. Entov, "Displacement of immiscible fluids during nonlinear percolation," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 2 (1968).
3. S. N. Buzinov, "Determination of residual petroleum saturation," Dokl. Akad. Nauk SSSR, 116, No. 1 (1957).